

# Instabilities and mode locking of quantum cascade lasers

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## ***Acknowledgement:***

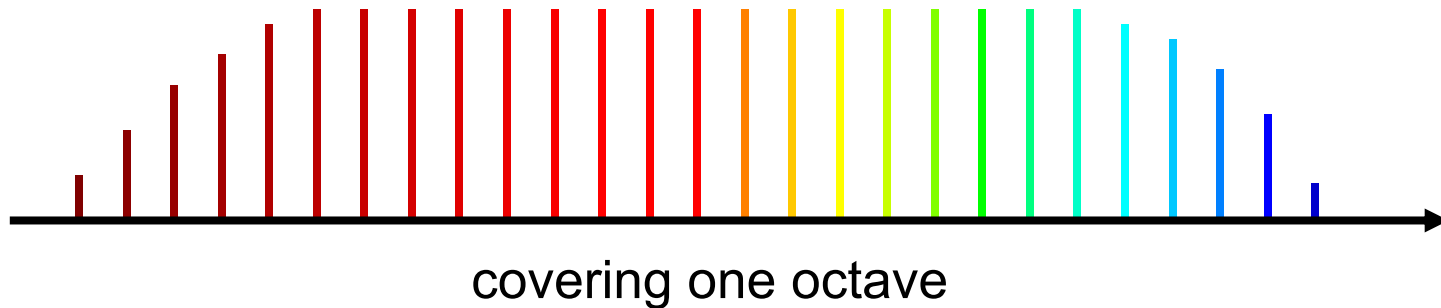
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# Outline

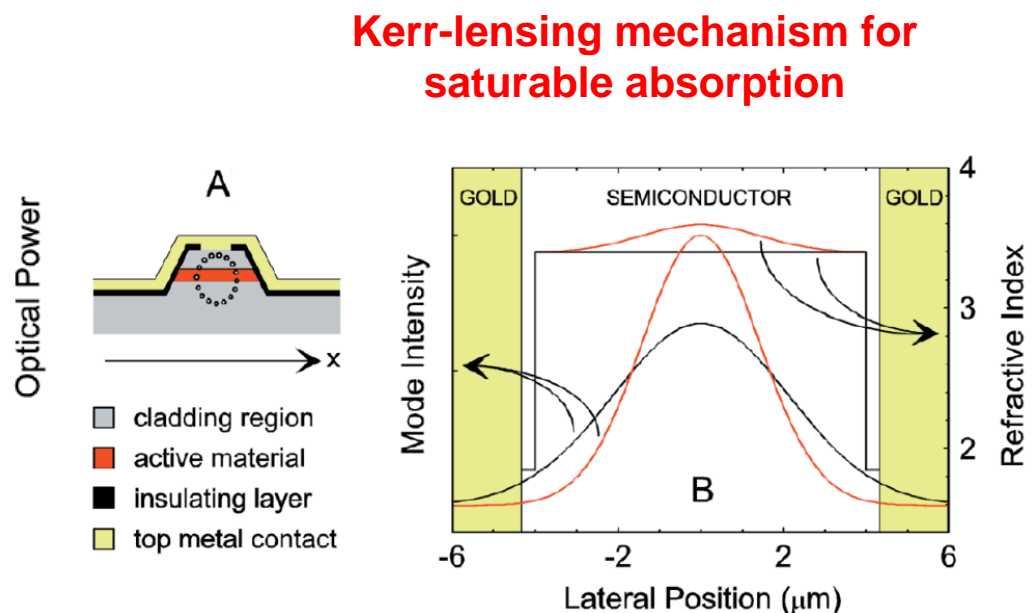
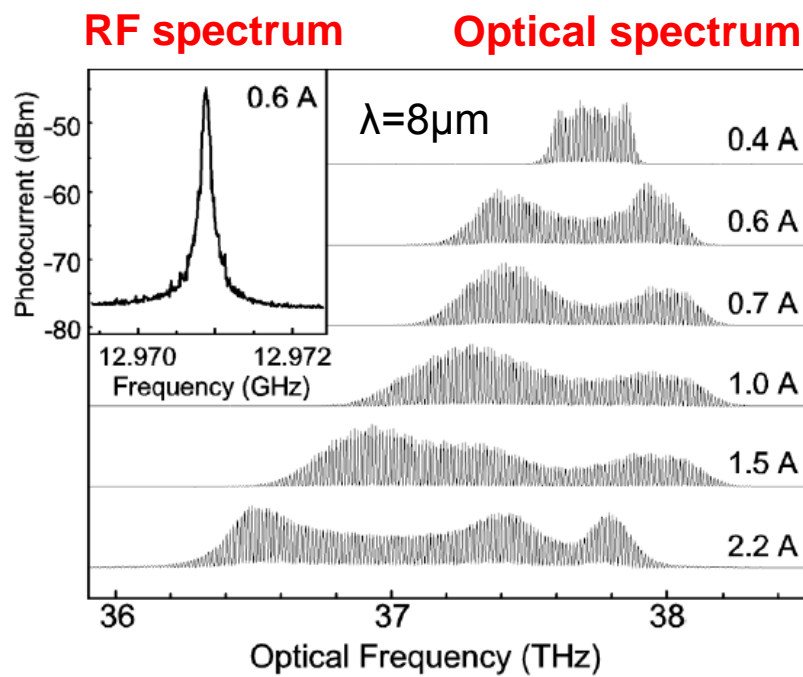
- Motivation and Experimental Observations
- What's different about QCLs compared to other Lasers?
- A simple two-level model for QCLs
- Multimode Instabilities:
  - Spectral Hole Burning
  - Risken-Nummedal-Graham-Haken Instability
- Actively Modelocked QCLs with Super Diagonal Gain
- Conclusion

# MID-IR Frequency Comb Spectroscopy: 2-5 $\mu\text{m}$ and 8-14 $\mu\text{m}$ Finger Print Regions



- A frequency comb from a stabilized modelocked laser is equivalent to an enormous number ( $\sim 100,000$ ) of narrow band cw laser.
- Provides amplitude and phase across hundreds of nanometers, which can simultaneously probe multiple transitions.
- Offers a frequency resolution and accuracy  $> 10^{-15}$

# First Evidence of Self Pulsation in QCLs

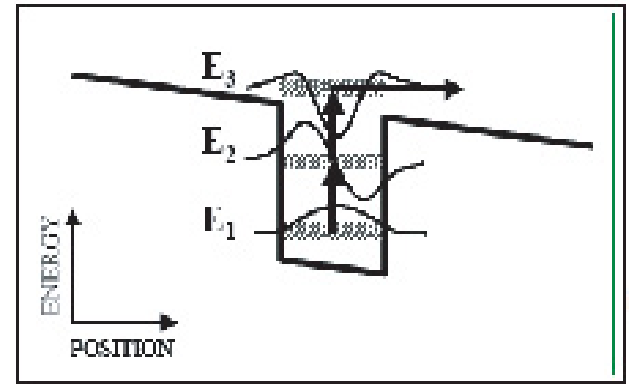
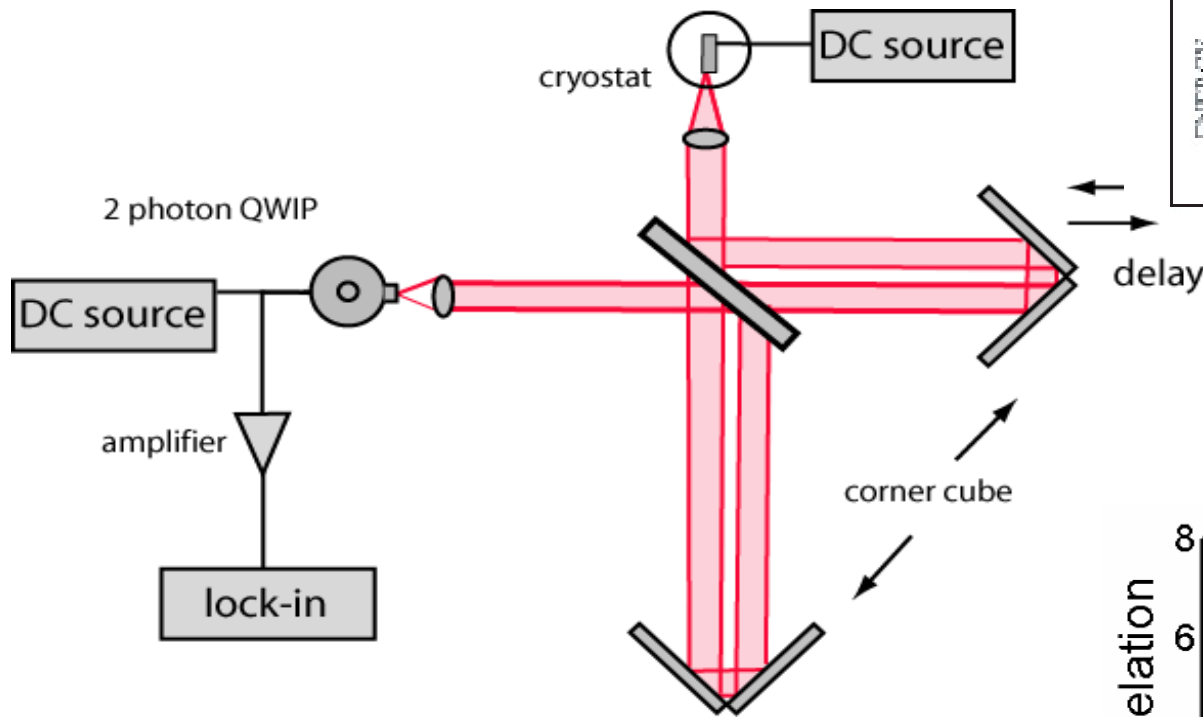


## What can we learn from the data?

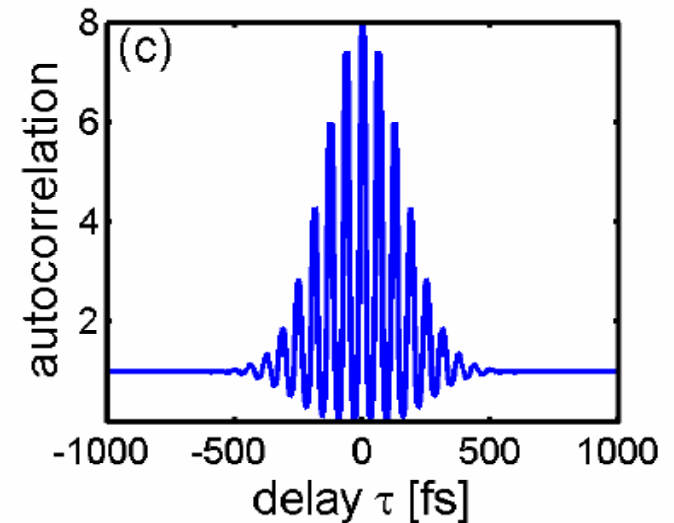
1. Many modes are lasing
2. The modes maintain coherence for  $\sim 1000^{\text{th}}$  roundtrips.

# Isolated Single Pulses ?

## Interferometric Autocorrelation

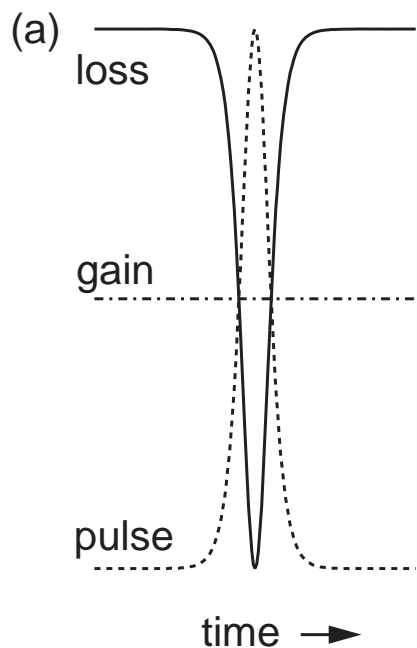


For isolated pulses  
Ratio < 8:1

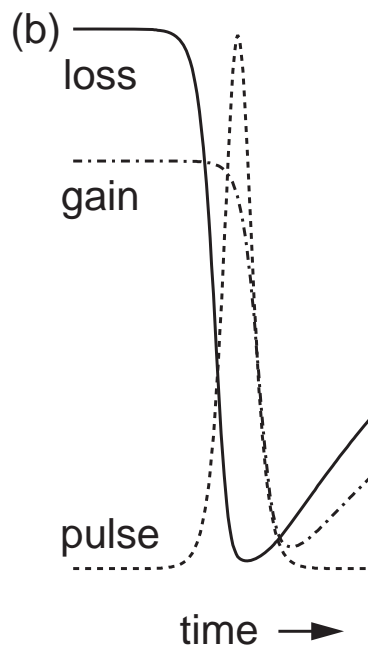


QWIPs from H.C. Liu and H. Schneider

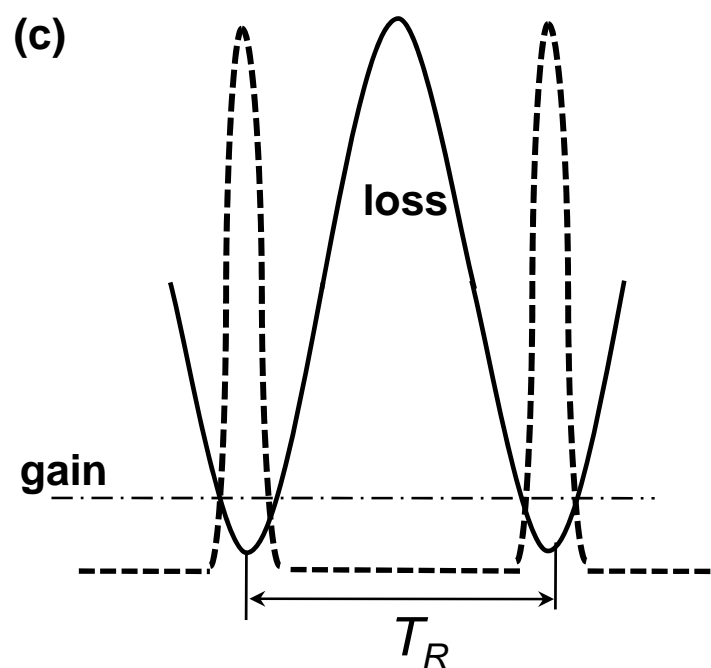
# Stable Modelocking Needs Net Gain Window



Solid-State  
Lasers



Semiconductor or  
Dye Lasers



Active Modelocking

Typical QCLs have 1ps-type gain recovery

Super-Diagonal QCLs → 50ps gain recovery

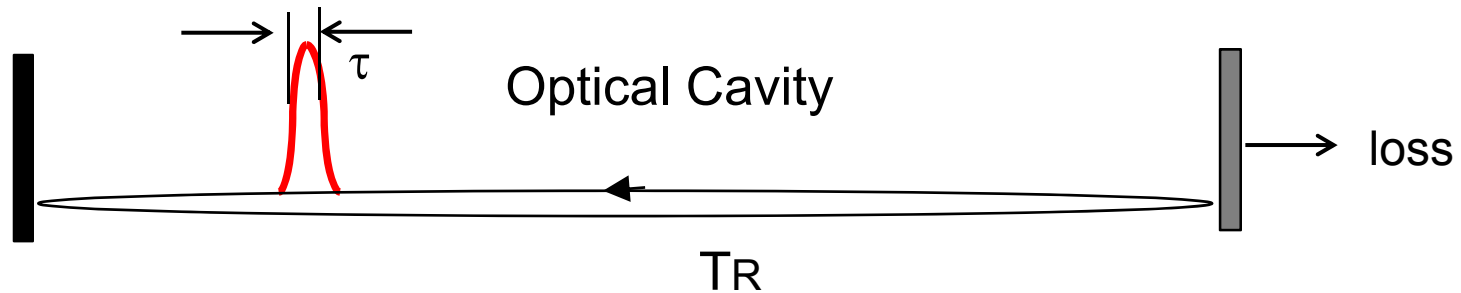
# Comparison of Different Laser Media

Medium	Upper State Lifetime $T_1$	Dipole Decay $T_2$	Gain sat. Intensity $I_s \sim T_1 \cdot T_2  \mu ^2$	Rabi Frequency $\Omega_{\text{Rabi}} = \frac{\mu E}{2\hbar}$	Mode Spacing $1/T_R$
Solid State	1 $\mu$ s-1 ms	10fs-1ps	~kW/cm <sup>2</sup>	10-100 MHz	10-100 MHz
Semi-conductor	>1 ns	50 fs	~kW/cm <sup>2</sup>	10 GHz	10-100 GHz
QCL	0.5 – 50 ps	100 – 500 fs	~100 kW/cm <sup>2</sup>	1THz	10-20 GHz

**Dipole dynamics important**

**Dipole dynamics couples many longitudinal modes**

# Maxwell – Bloch Equations



## Polarization

$$\dot{\rho}_{ab} = i\omega\rho_{ab} + i\frac{\mu E}{\hbar}\Delta - \frac{\rho_{ab}}{T_2},$$

## Diffusion

## Inversion

$$\dot{\Delta} = -2i\frac{\mu E}{\hbar}(\rho_{ab}^* - \rho_{ab}) - \frac{\Delta_p - \Delta}{T_1} + D\frac{\partial^2 \Delta}{\partial z^2}$$

## Field

$$\partial_z^2 E - \frac{n^2}{c^2}\partial_t^2 E = \frac{N\Gamma\mu}{\epsilon_0 c^2}\partial_t^2 (\rho_{ab} + \rho_{ab}^*).$$



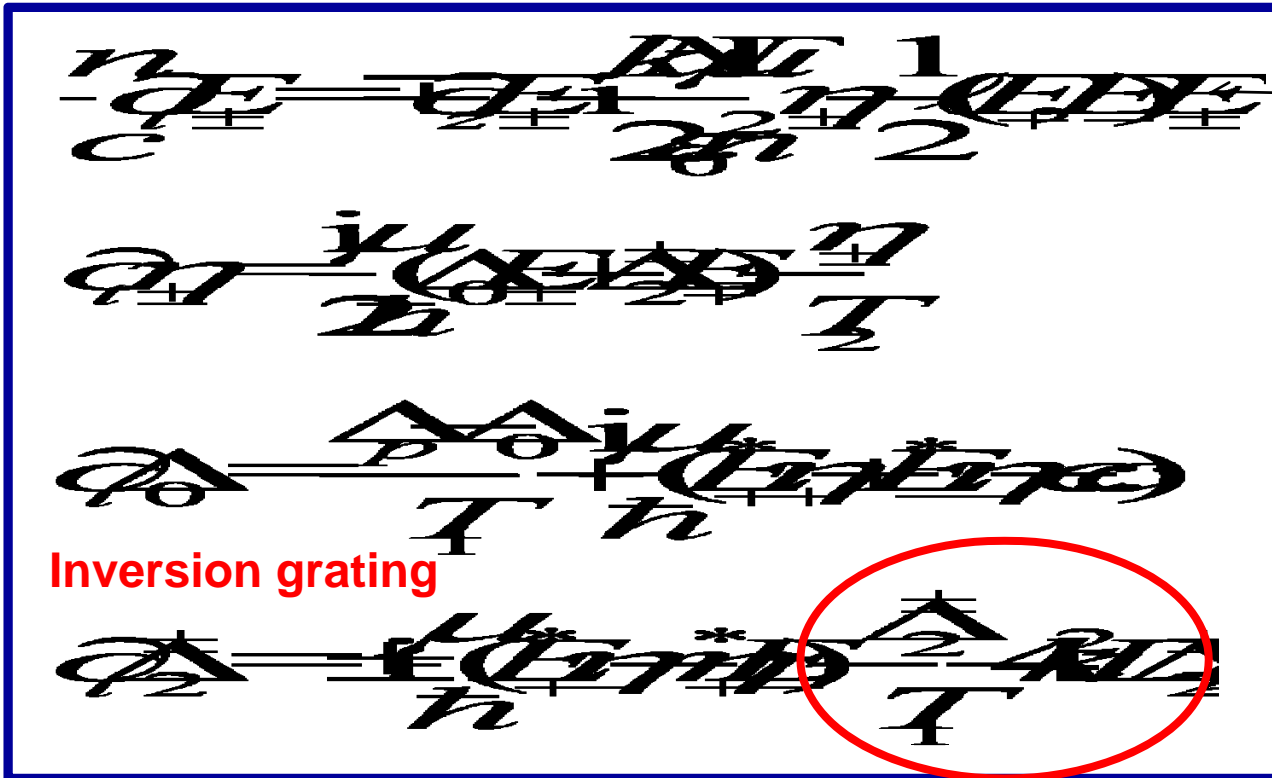
# Inversion Grating

$$E(z,t) = \frac{1}{2} \left[ E_+^*(z,t) e^{-i(\omega t - kz)} + E_+(z,t) e^{i(\omega t - kz)} \right] + \frac{1}{2} \left[ E_-^*(z,t) e^{-i(\omega t + kz)} + E_-(z,t) e^{i(\omega t + kz)} \right]$$

$$\rho_{ab}(z,t) = \eta_+(z,t) e^{i(\omega t - kz)} + \eta_-(z,t) e^{i(\omega t + kz)}$$

$$\Delta(z,t) = \Delta_0(z,t) + \Delta_2(z,t) e^{2ikz} + \Delta_2^*(z,t) e^{-2ikz},$$

$$k = \frac{2\pi}{\lambda_L}$$



Saturable Absorber Effects

$$l = \gamma |E|^2$$

Inversion Grating Decay Time

$$\frac{1}{\tau} = \frac{1}{T} + \dots$$

Grating Strength

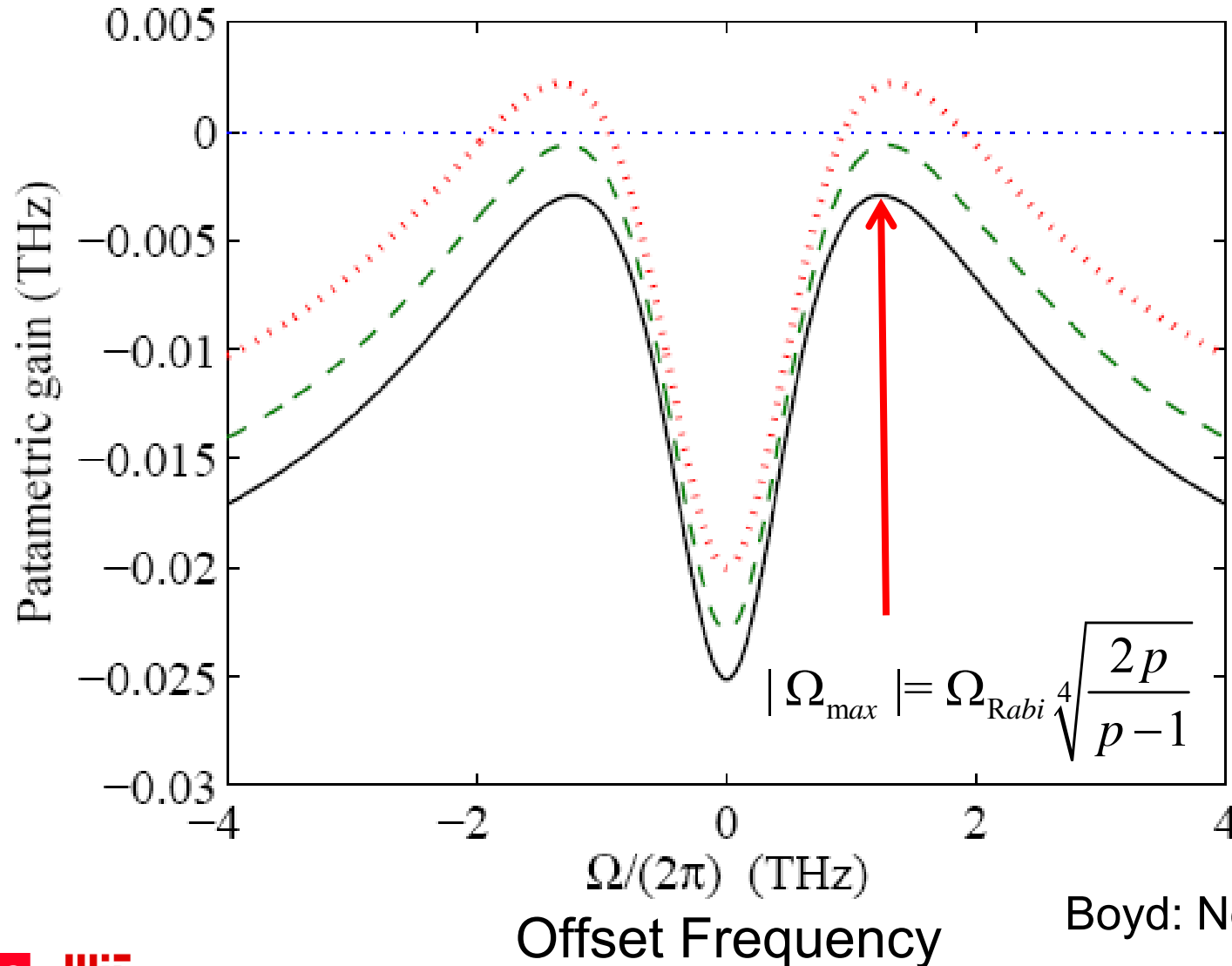
$$\frac{1}{1 + 4k^2 D_L^2}$$

# **Spatial Hole Burning Effects**

**Strong in QCLs**

# Ring Cavity: (Riskin Nuhmedal Graham & Haken Only)

- No Standing Waves → No Spatial Holeburning



Pump  
Parameter

$$p \equiv \frac{\Delta_p}{\Delta_{th}}$$

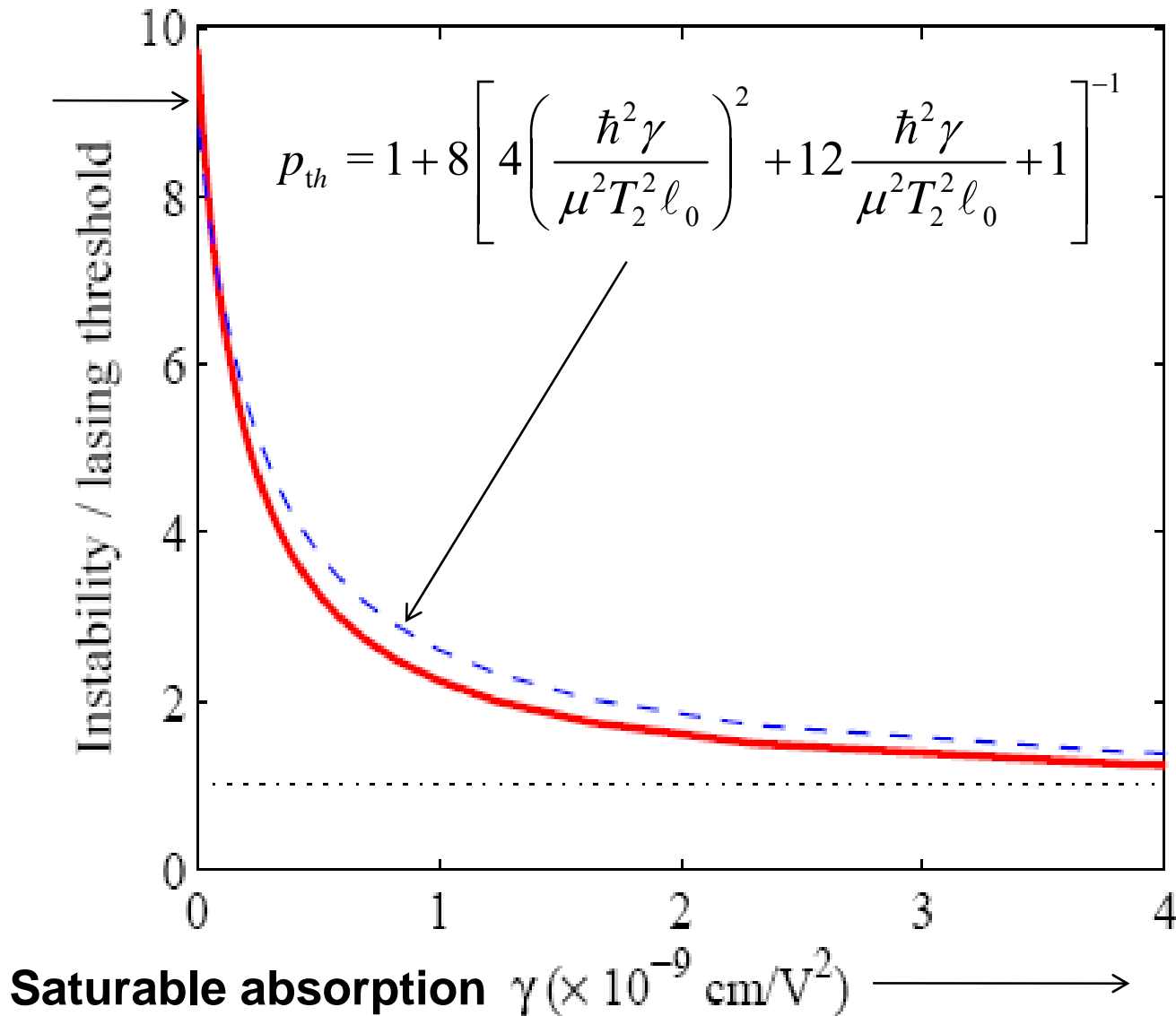
CW- Rabi  
Frequency

$$\Omega_{Rabi} = \sqrt{\frac{p-1}{T_1 T_2}}$$

Boyd: Nonlinear Optics

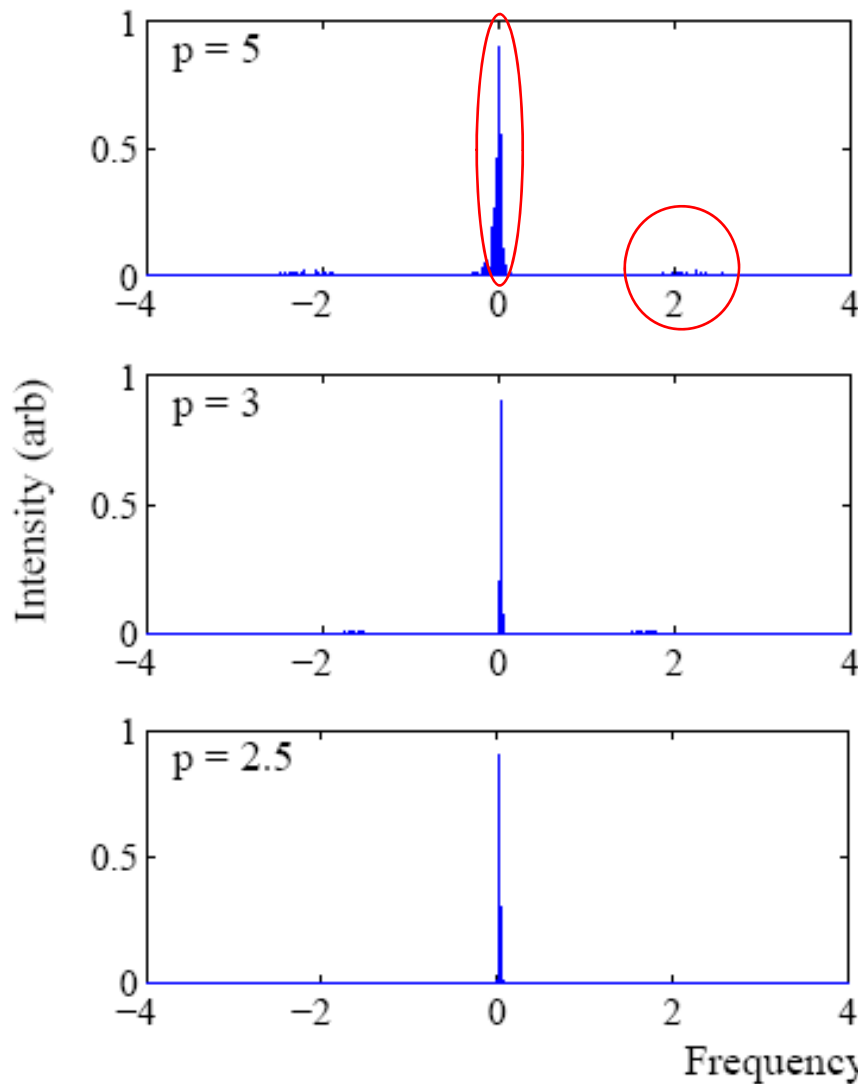
# RNGH Instability Threshold

Original  
RNGH  
1968

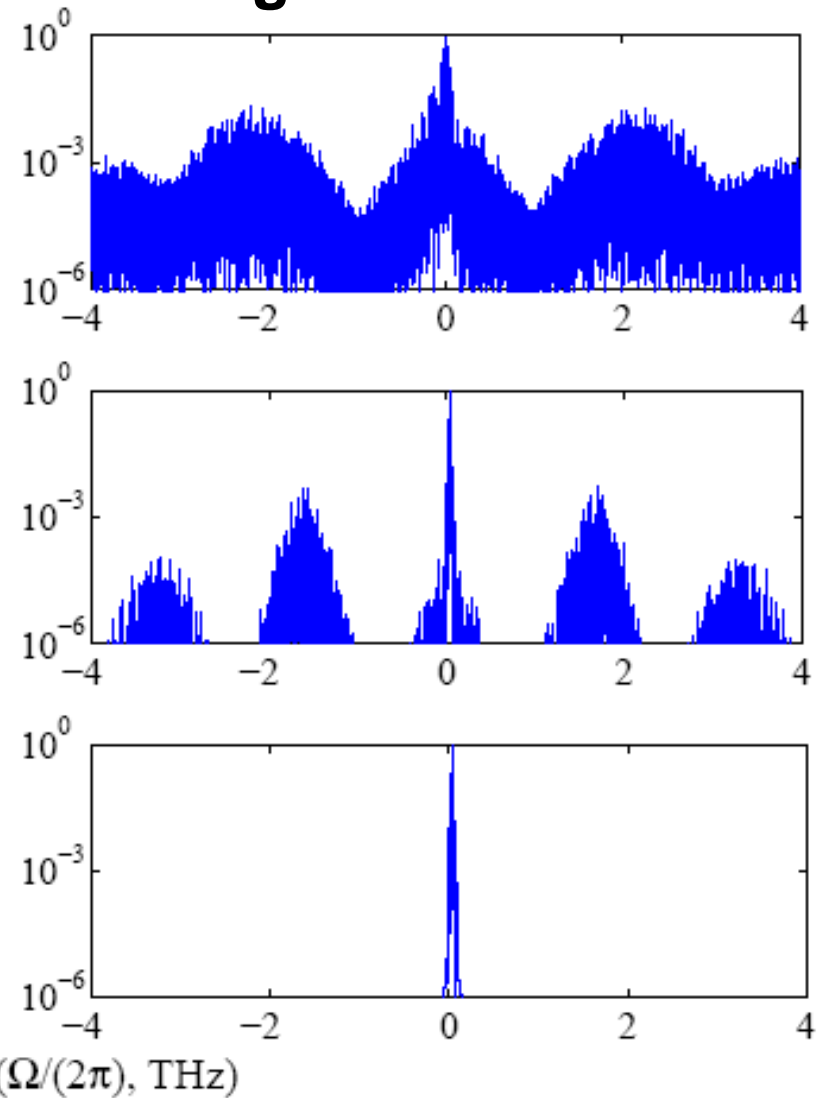


# Numerical Simulations

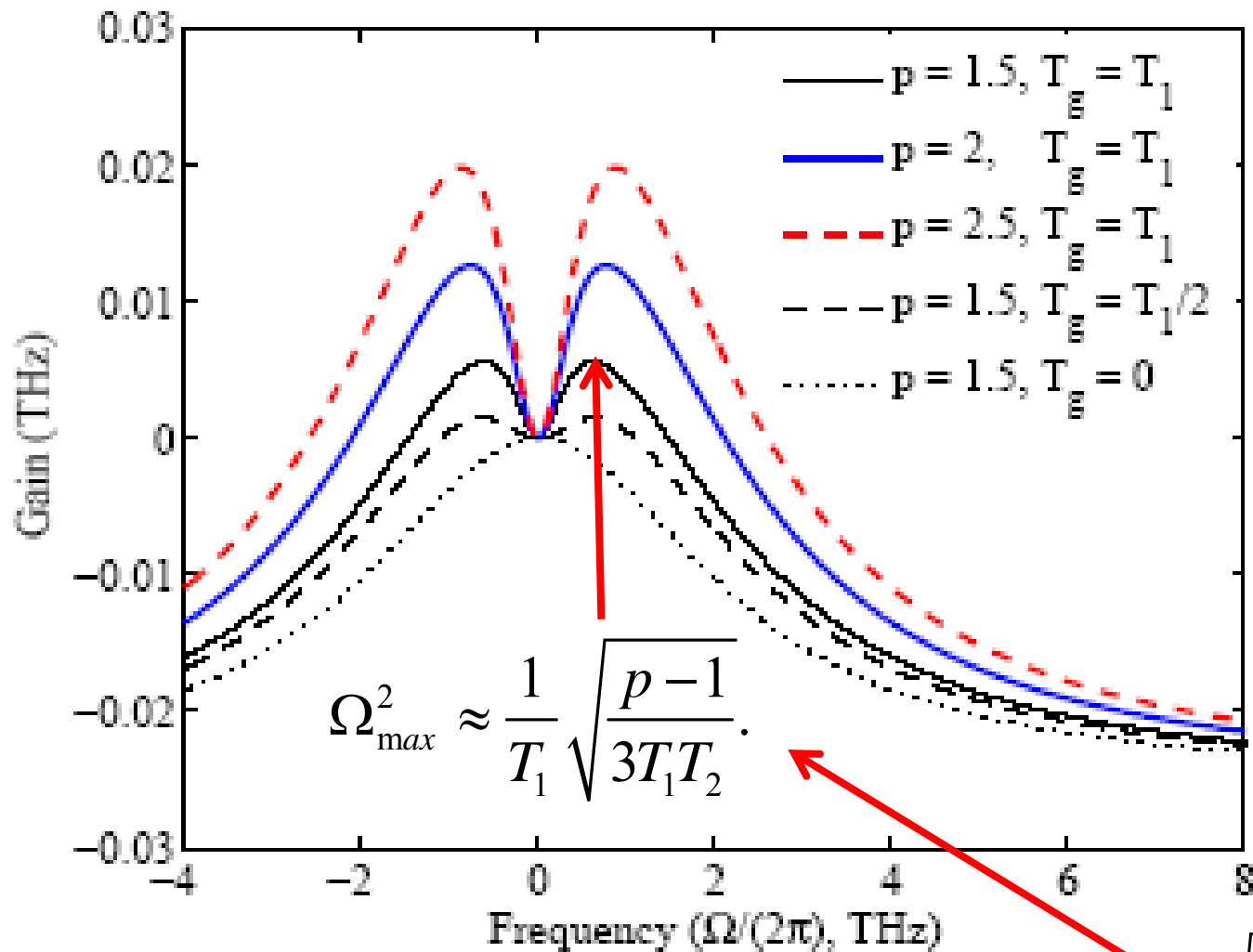
## Linear Scale



## Logarithmic Scale



# Linear (Standing Wave) Cavity (SHB)



Inversion Grating  
Decay Time



AllnAs-nGaAs  
heterost.

$$D = 400 \text{ cm}^2/\text{sec}$$

$$k = 3.7 \times 10^4 \text{ cm}^{-1}$$

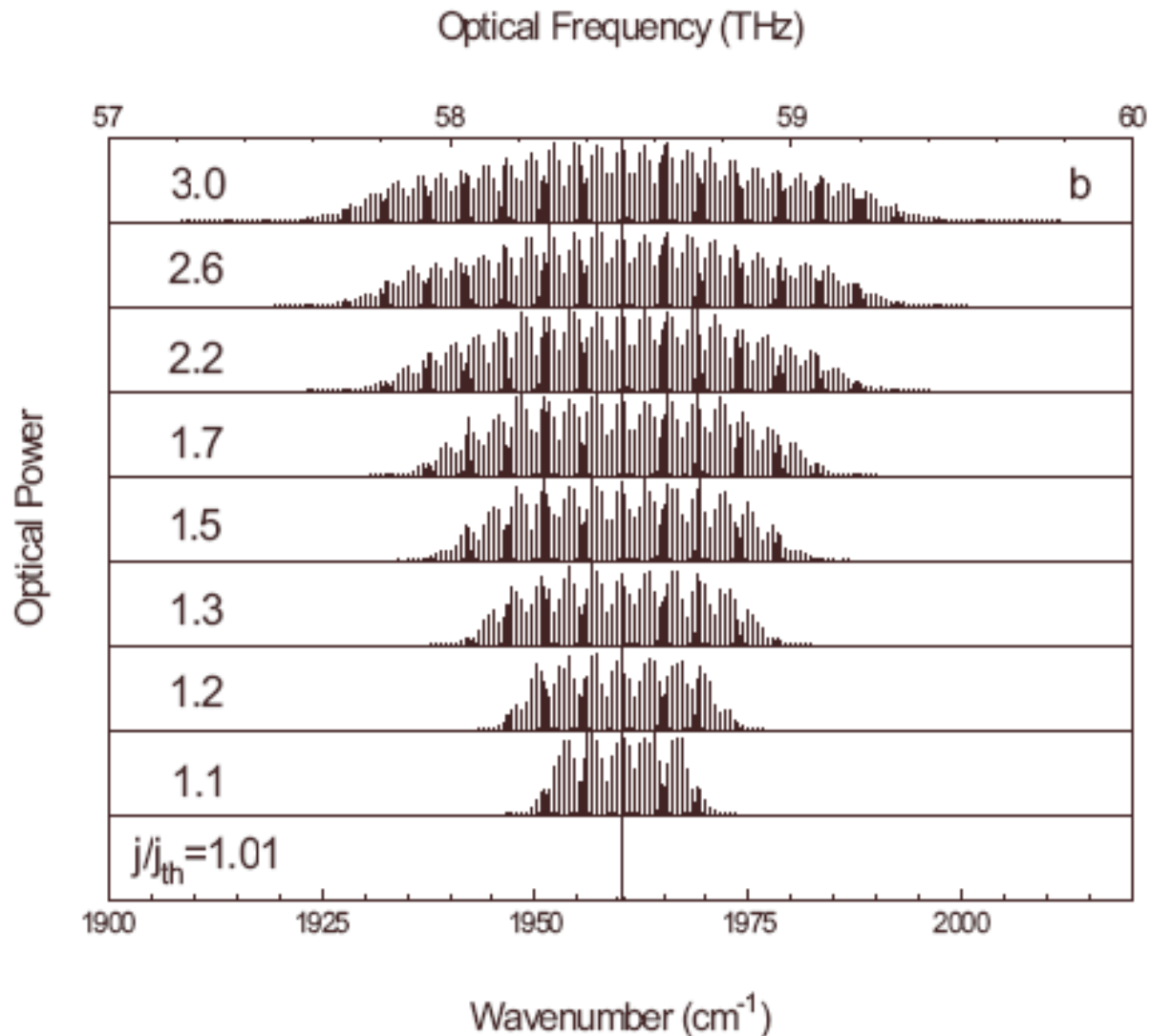
$$4k^2D \approx 2 \text{ THz}$$

$$\sqrt{\frac{1}{T_1} \cdot \Omega_{\text{Rabi}}}$$

# Parameter Values

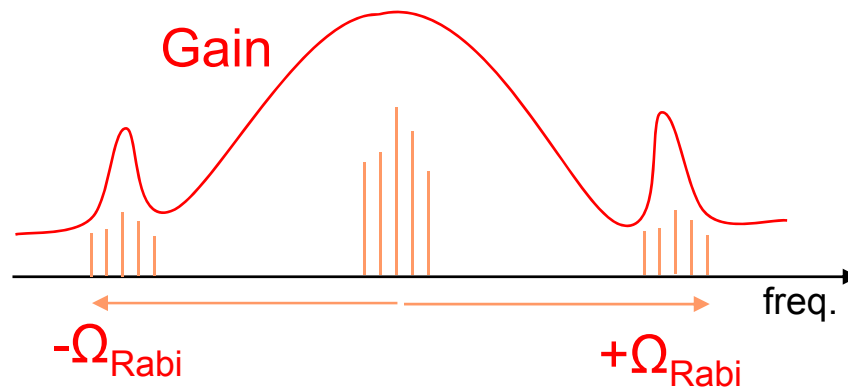
Quantity	Symbol	Value
Gain recovery time	$T_1$	0.5 ps
Dephasing time	$T_2$	0.067 ps
Linear cavity loss	$l_0$	$5 \text{ cm}^{-1}$
Transition dipole element	$\mu$	$2.54 \text{ nm} \times e$
Background refractive index	$n$	3
Cavity length	$L$	3 mm
Saturable absorber coefficient	$\gamma$	$10^{-8} \frac{\text{cm}}{\text{V}^2}$

# Numerical Sim.: SHB, RNGH but no saturable absorber

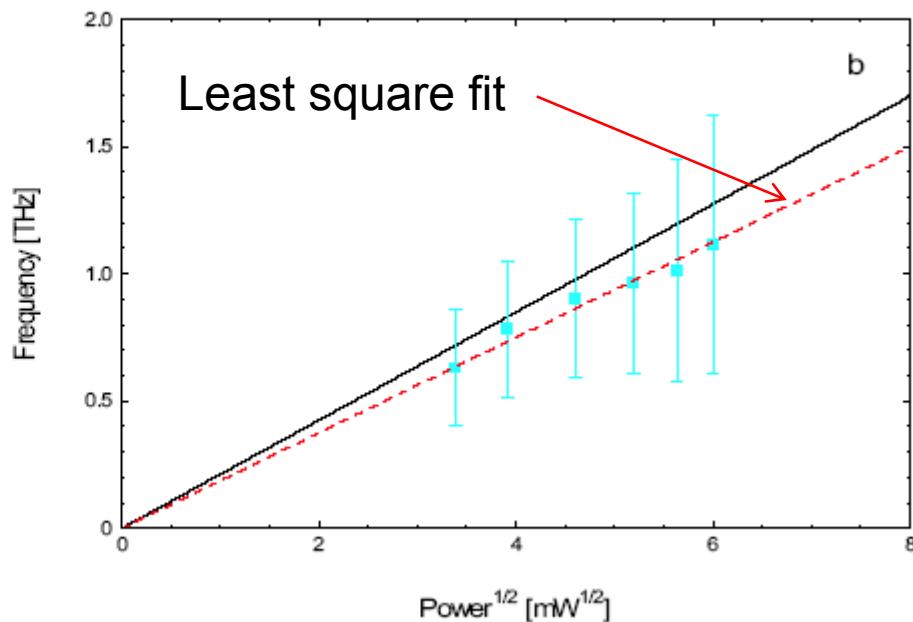
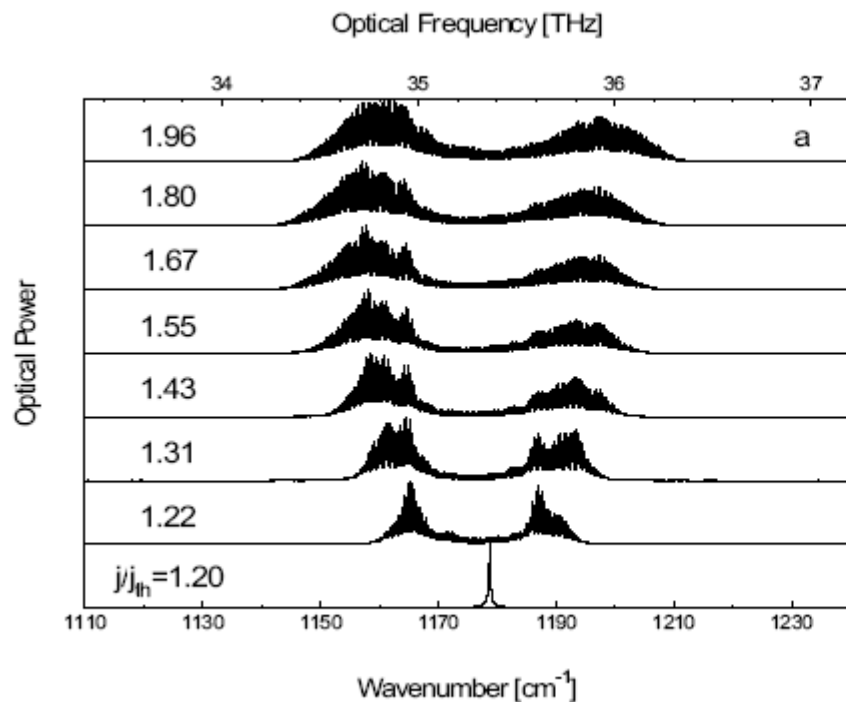




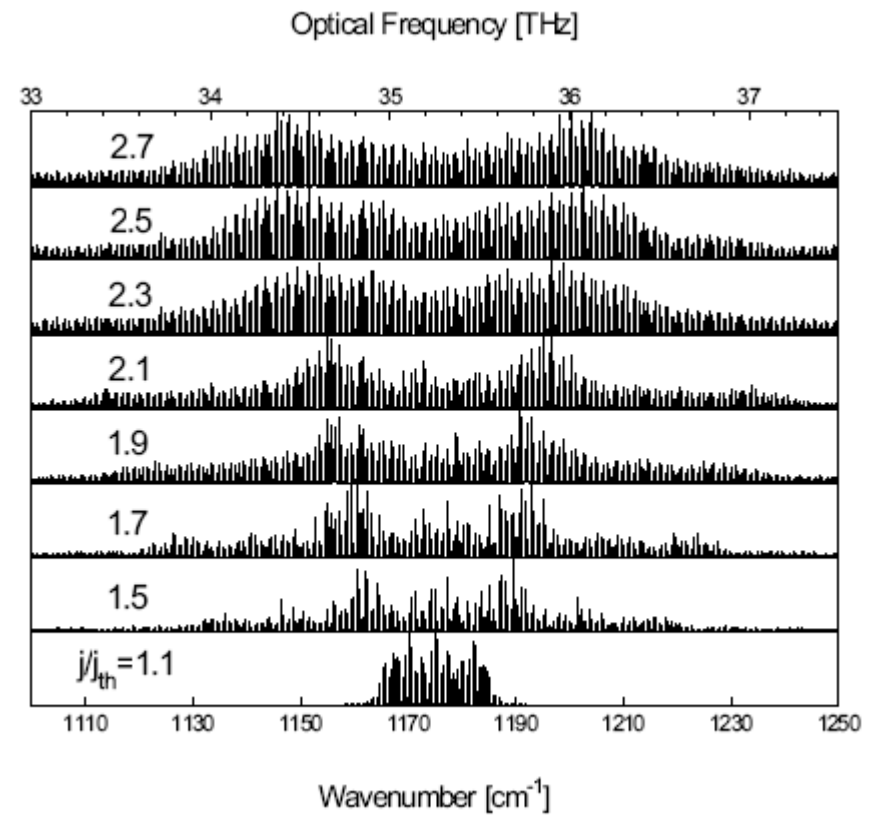
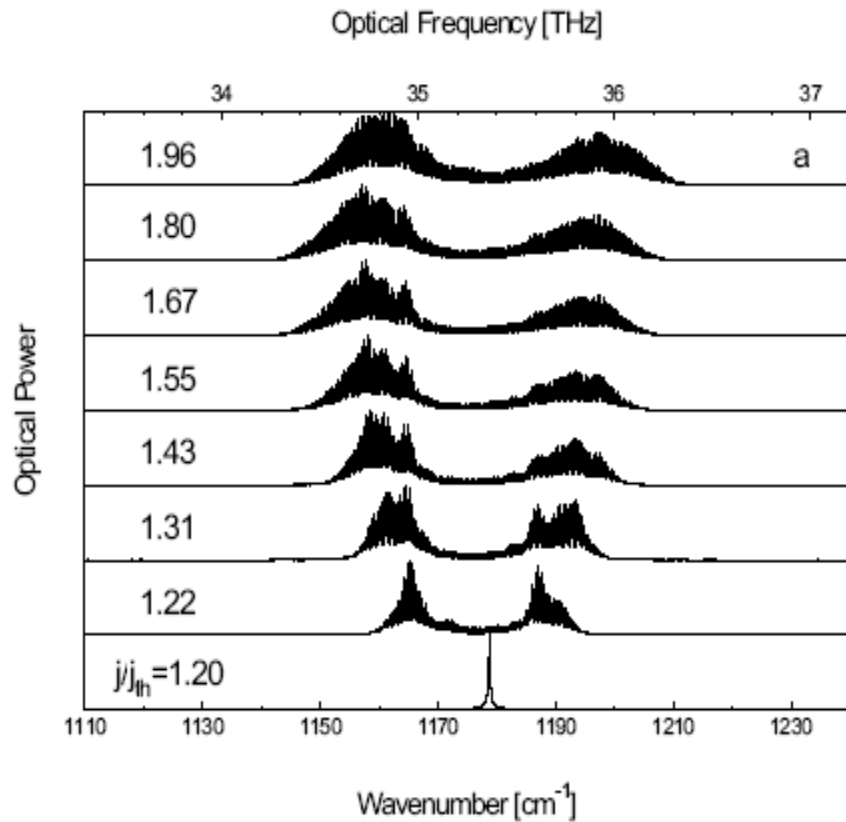
# Including Saturable Absorber



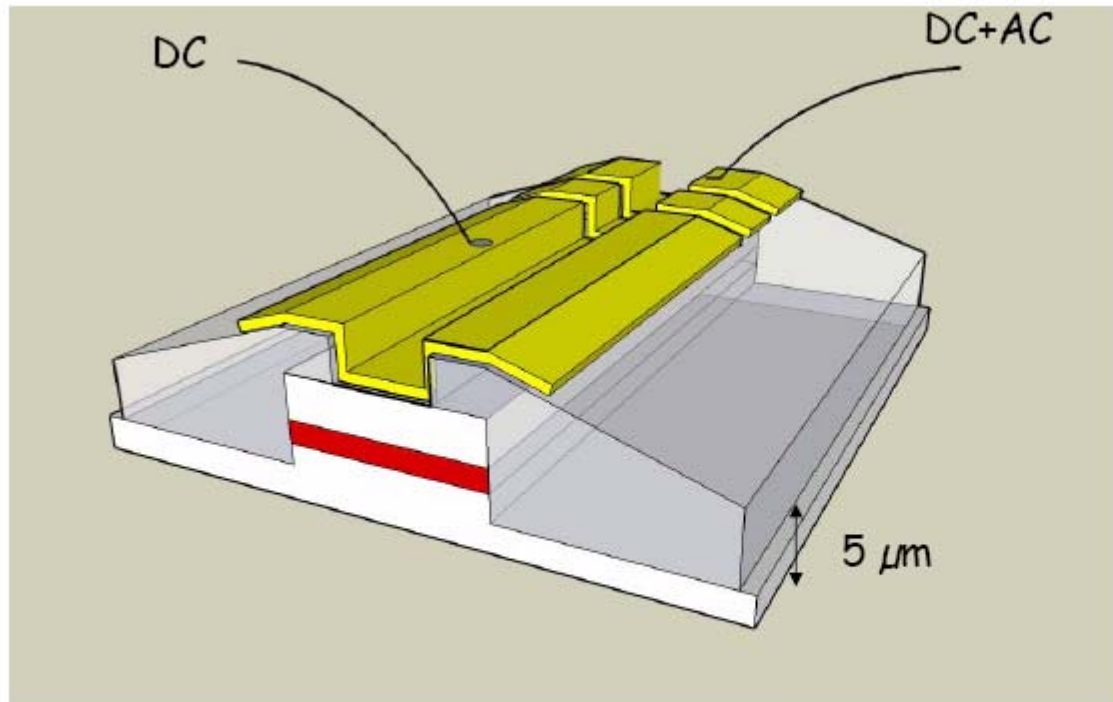
3  $\mu\text{m}$  wide buried heterostructure laser at 8.38  $\mu\text{m}$ .



# Maxwell-Bloch Simulations



# Actively Modelocked QCL



Super diagonal gain  
 $T_1 = 50\text{ps}$

Thick insulation is used to reduce the parasitic capacitance.

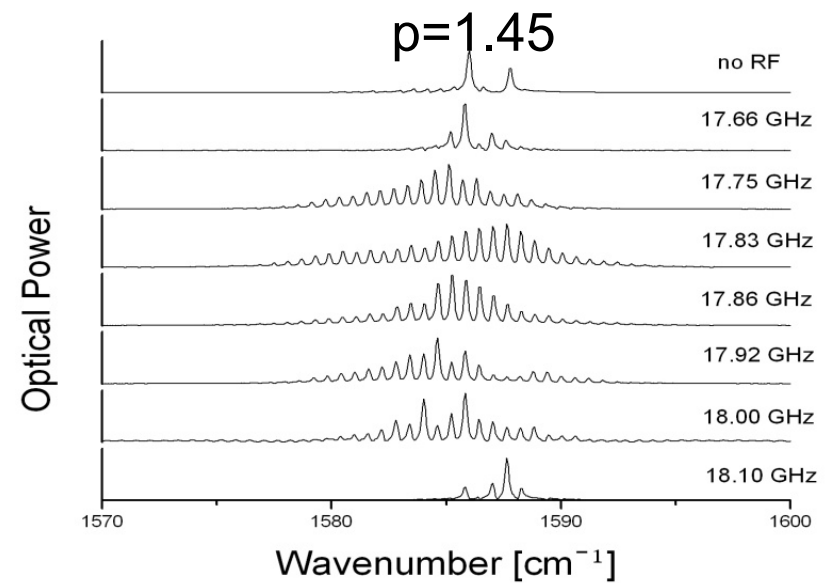
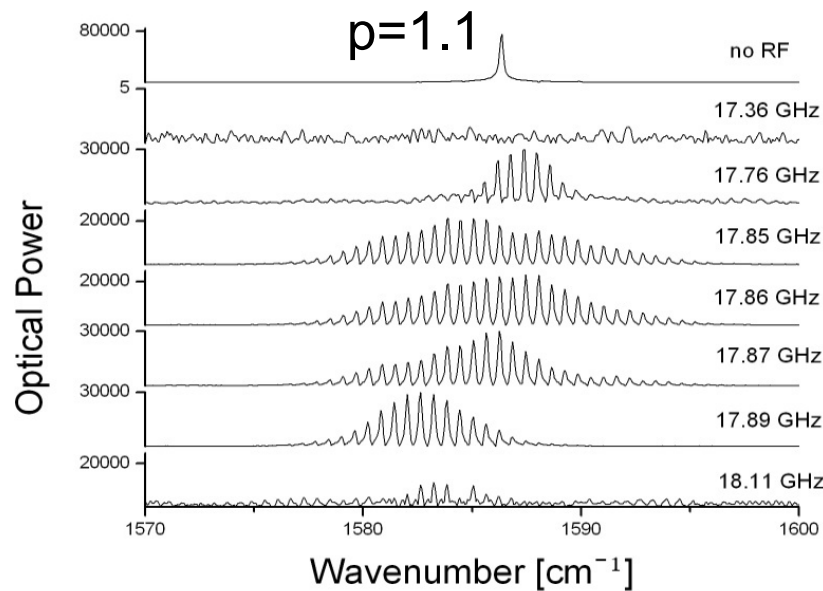
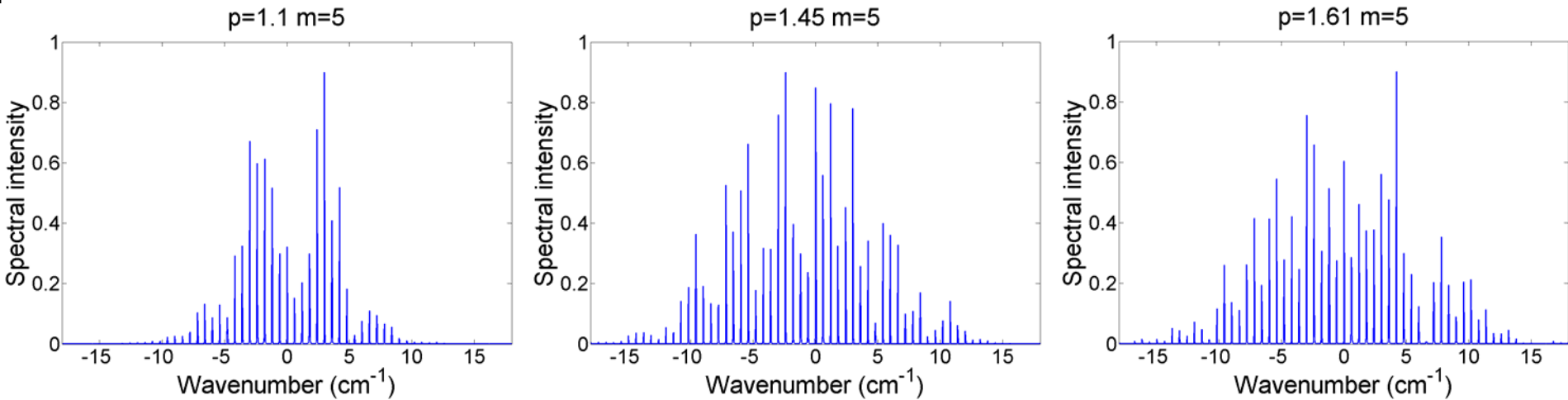
Pumping of small section:  $\lambda = p * \lambda_{\text{th}} + m * \lambda_{\text{th}} * \sin(\omega_R * t)$

Pumping of long section:  $\lambda = p * \lambda_{\text{th}}$

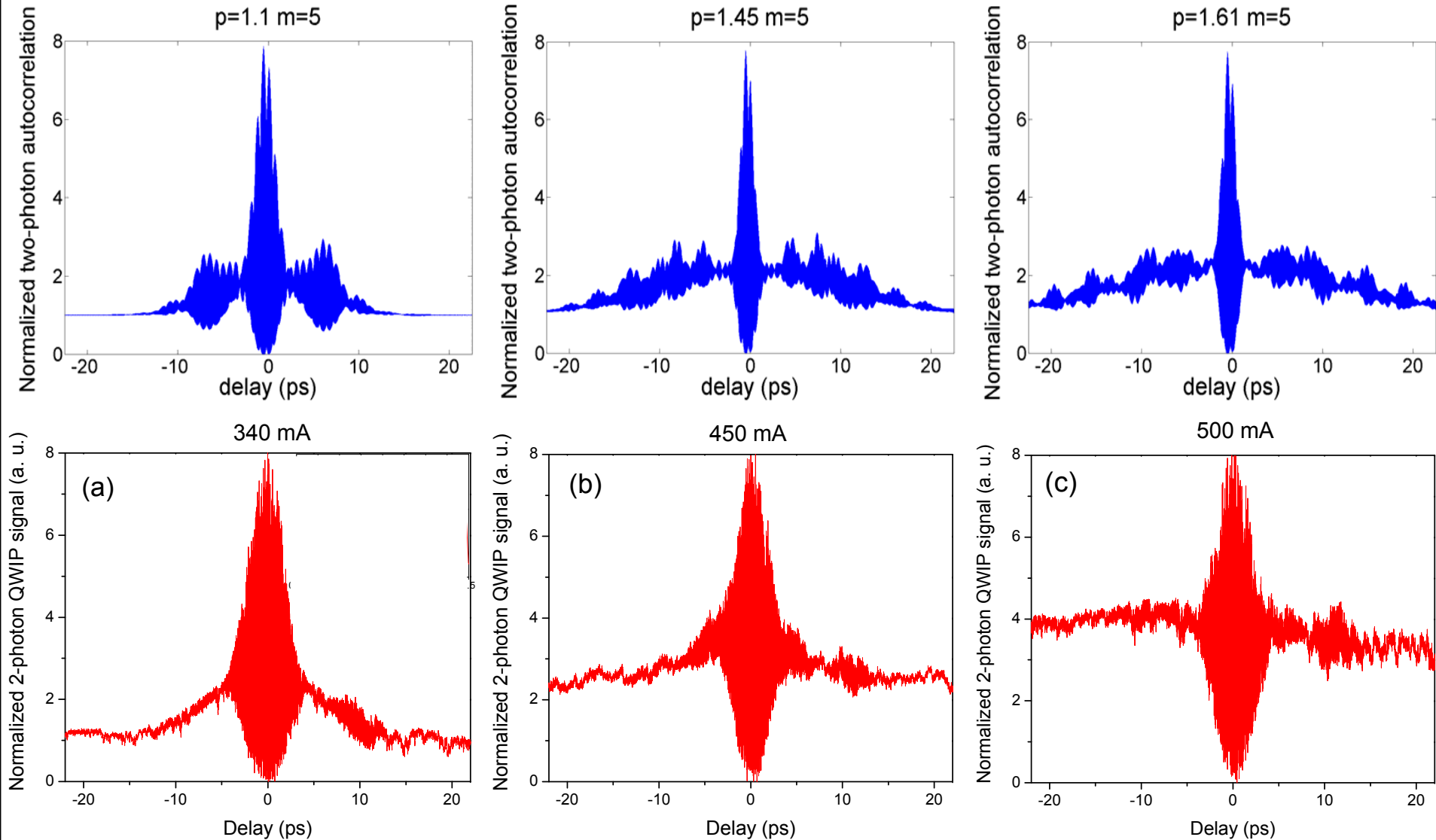
# Parameter values

Quality	Symbol	Value
Gain Recovery Time	$T_1$	50 psec
Dephasing time	$T_2$	0.05 psec
Linear cavity loss	$l_0$	10 cm <sup>-1</sup>
Mirror reflectivity	R	0.53
Transition dipole element	$\mu$	2.54 nm $\times$ e
Background refractive index	$n$	3.2
Cavity length	$L$	2.6 mm
Modulator section length	$L_s$	0.24 mm

# Spectra

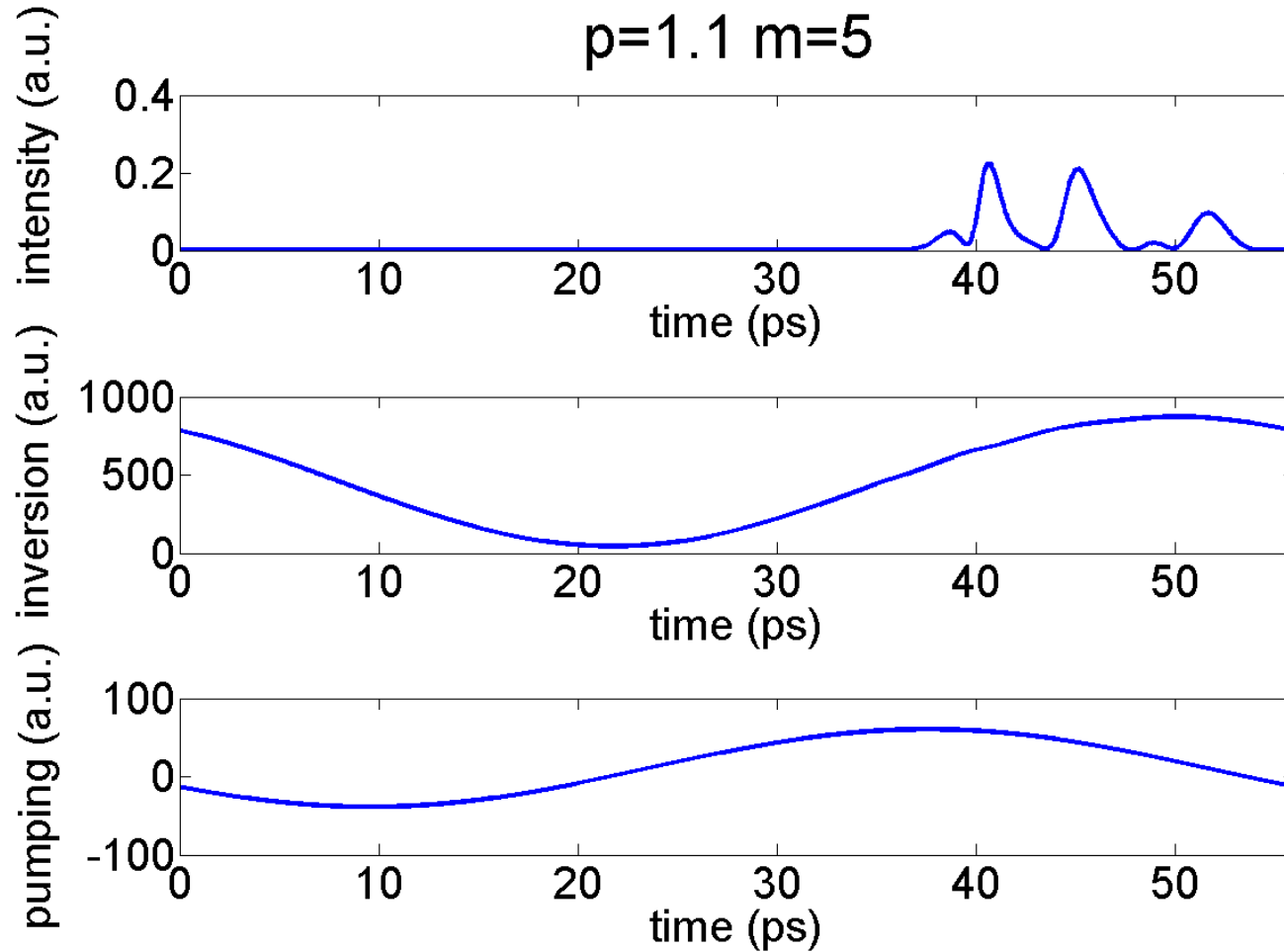


# Autocorrelation plots including SHB



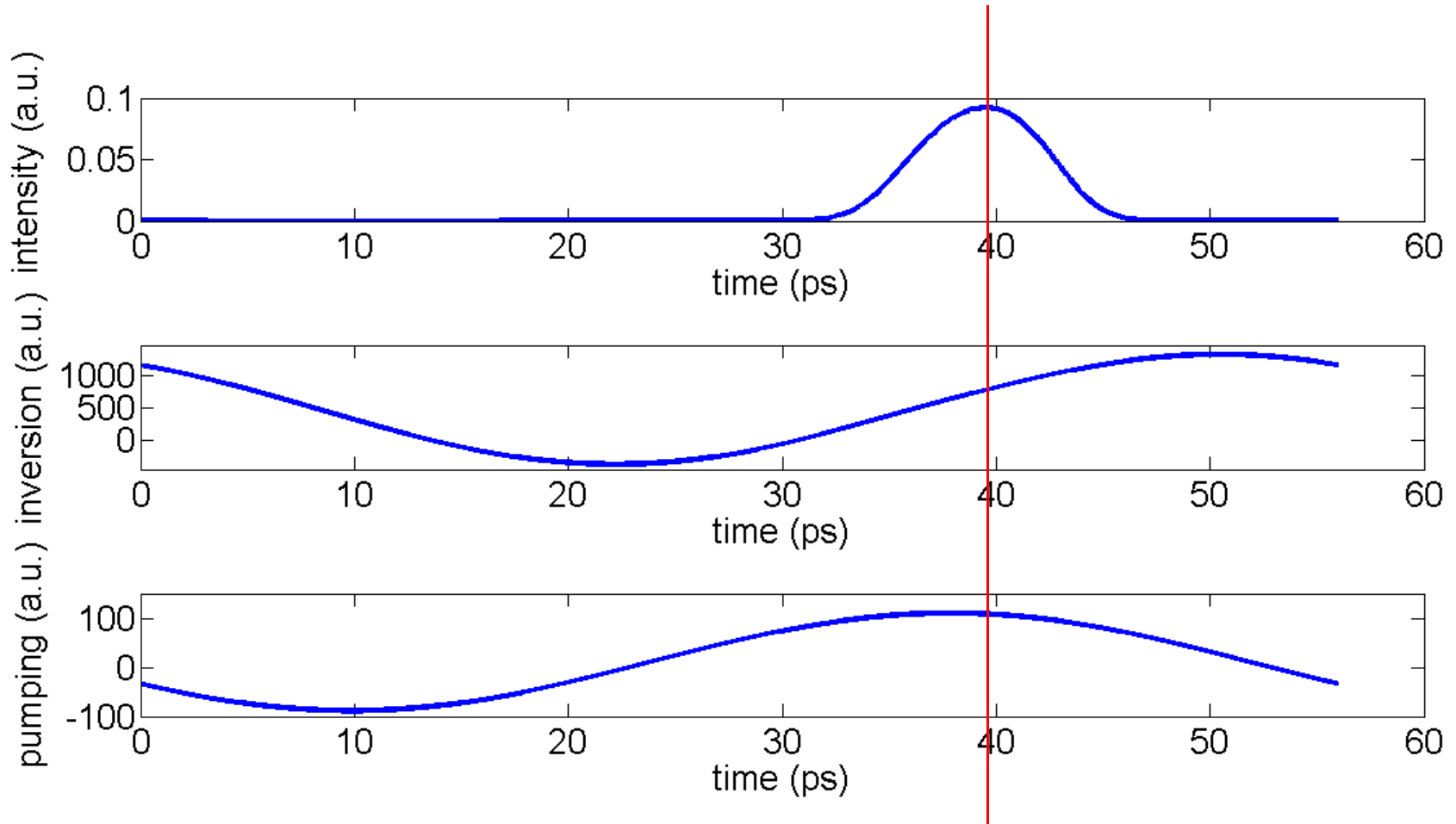
# Laser Dynamics

Inside the modulation region:



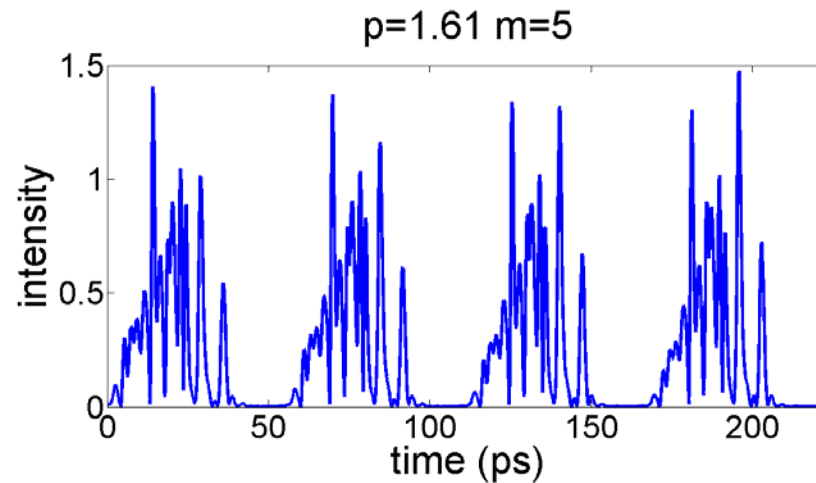
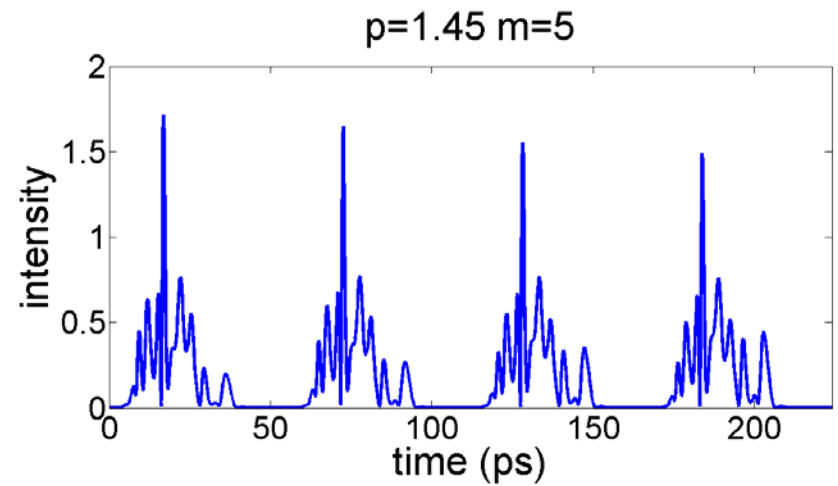
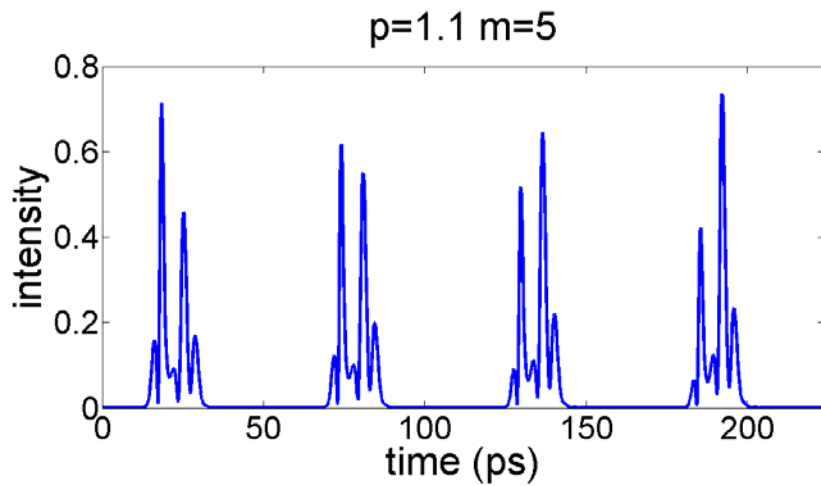
# Laser Dynamics

Inside the modulation region without SHB:





# Pulse Trains at Output with SHB



# Summary

## 1. Short gain recovery time

- Enhances spectral holeburning – wideband multimode operation
- No stable modelocking

## 2. Including saturable absorber effects

- RNGH like coherent instability
- Explains double-humped spectral shape
- A ring laser suppressing SBH could confirm this picture

## 3. Routes to stable short pulses:

- a. Long gain recovery – superdiagonal structure  
Active modelocking leads to isolated “modelocked” pulses.
- b. Next: Passive mode locking with reverse biased section